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Radiation Energy Flux and Radiation Power of Schwarzschild Black Hole

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Abstract Applying the entropy density near the event horizon, we obtained the result that the radiation energy flux of the black hole is always proportional to the quartic of the temperature of its event horizon. That is to say, the thermal radiation of the black hole always satisfies the generalized Stefan–Boltzmann law. The derived generalized Stefan–Boltzmann coefficient is no longer a constant. When the cut-off distance and the thin film thickness are both fixed, it is a proportional coefficient which is related to the black hole mass, the kinds of radiation particles and space–time metric near the event horizon. In this paper, we have put forward a thermal particle model in curved space–time. By this model, the result has been obtained that when the thin film thickness and the cut-off distance are both fixed, the radiation energy flux received by observer far away from the Schwarzschild black hole is proportional to the average radial effusion velocity of the radiation particles in the thin film, and inversely proportional to the square of the distance between the observer and the black hole.

Keywords Schwarzschild black hole · Average radial effusion velocity · Radiation energy flux · Radiation power

1 Introduction

Since Bekenstein and Hawking suggested that the entropy of the black hole is proportional to its area of the event horizon [1, 2], many methods have been put forward to calculate the black hole entropy [3–6]. For example, the brick-wall model proposed by 't Hooft gives a statistical explanation to the origin of the black hole entropy [3]. By this model, the entropy of static and stationary black holes is calculated out separately, and the same result that the proportionality between entropy and area is obtained. Because the Hawking radiation originates from the vacuum fluctuation near the event horizon, the brick-wall model has been developed to the thin film model [7–10]. According to this thin film model, the entropy of

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the black hole comes from the contribution of quantum field in an infinitesimal thin film near its event horizon. Many researchers have adopted the thin film model to calculate the black hole entropy, and obtained the same result that black hole entropy is proportional to the area of the event horizon [11–13]. Where the event horizon is, there are black hole entropy and Hawking radiation [14]. In 2000, considering the self-gravitation action of the radiation particles and regarding the Hawking radiation as a quantum tunnelling process, Parikh and Wilczek obtained that the quantum tunnelling rate is related to the change of the entropy of Bekenstein-Hawking [15]. Then some authors [16–20] have obtained the same result as that of Parikh. It is shown that there must be an intrinsic relation between the entropy and the thermal radiation of the black hole. Therefore, it is of significance to further study this intrinsic relation. Recently, we have studied the thermal radiation of the black hole by the entropy density near its event horizon, and found that the thermal radiation of the black hole satisfies the generalized Stefan–Boltzmann law [21–28]. The corresponding generalized coefficient is no longer a constant, but a coefficient related to the space–time metric around the black hole. So the thermal radiation of the curved space–time is different from that of the flat space–time. The strong gravitational field around the black hole will affect its thermal radiation. In order to make these results more universally significant, by adopting the thin film model, this paper studied the radiation energy flux and radiation power, and discovers the intrinsic relation between gravitational field around the black hole and its thermal radiation.

2 Generalized Stefan–Boltzmann law for Schwarzschild black hole

The line element of the Schwarzschild black hole is given by

$$\mathrm{d}s^2 = -\left(1-\frac{2M}{r}\right)\mathrm{d}t^2 + \left(1-\frac{2M}{r}\right)^{-1}\mathrm{d}r^2 + r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2), \tag{1}$$

where M is the black hole mass. The position of its event horizon is

$$r_H = 2M, \tag{2}$$

and its temperature is

$$T = \frac{1}{\beta} = \frac{1}{8\pi M}. \tag{3}$$

By the thin film model, the entropy of scalar field of the Schwarzschild black hole is [10]

$$S = \frac{128\pi^3 M^4}{45\beta^3}\frac{\delta}{\varepsilon(\varepsilon+\delta)}, \tag{4}$$

where ε is the cut-off distance and δ is the thin film thickness. If we let $\delta = n\varepsilon$, from (4), we can obtain

$$S_n = \frac{128\pi^3 M^4}{45\beta^3}\frac{n}{\varepsilon(n+1)}. \tag{5}$$

When $n \to \infty$, the corresponding entropy can be denoted by S_∞. From (5), one can obtain $S_n = \frac{n}{n+1}S_\infty$. If the cut-off distance is set as the Planck length (same to the 't Hooft intrinsic thickness), that is, $\varepsilon = l_p$, and when $\delta = 10l_p$, the derived entropy ($S_{10} = \frac{10}{11}S_\infty$) is approximately equal to the whole entropy of the black hole. Therefore, the black hole entropy mainly comes from the contribution of quantum field in a very thin film near its event horizon. According to the thin film model, the entropy density of scalar field near the event horizon can be obtain from (4)

$$s = \frac{S}{V} = \frac{8\pi^2 M^2}{45\beta^3 \varepsilon(\varepsilon + \delta)}, \tag{6}$$

where V is the volume of the thin film near the event horizon. Since the value of r_H is much greater than that of ε and δ, the volume of the thin film can be adopted as $V = 4\pi r_H^2 \delta$.

Based on the local equivalence principle, in the local area of the infinitesimal thin film near the event horizon, the basic equations of thermodynamics still hold. The energy density ρ and the entropy density s in the thin film near the event horizon, and the temperature in local area T satisfy the following relation [21]

$$\rho = bT^4, \tag{7}$$

$$s = \frac{4}{3} bT^3. \tag{8}$$

Combining (3), (6) and (8), one can get

$$b = \frac{6\pi^2 M^2}{45\varepsilon(\varepsilon + \delta)}. \tag{9}$$

Substituting (9) into (7), one can obtain

$$\rho = \frac{6\pi^2 M^2}{45\varepsilon(\varepsilon + \delta)} T^4. \tag{10}$$

It can be seen that, for a black hole, the energy density of scalar field of the thin film near its event horizon is proportional to the quartic of the temperature of its event horizon when the cut-off distance and the thickness of thin film are fixed. Considering the physical mechanism of Hawking radiation of the black hole, due to the virtual particle pair caused by the vacuum fluctuation near the event horizon, the energy of the black hole will decrease when the virtual particles with negative energy go back to the black hole by the tunnelling effects. At the same time, the virtual particles with positive energy will emit out of the gravitational area of the black hole and fly far away to form Hawking radiation. In fact, the motion of the particles with positive energy is very complex in the thin film ($r_H + \varepsilon \to r_H + \varepsilon + \delta$). The world line of the particles of zero rest mass is kind of light, while that of the particles of rest mass is kind of time. In order to make this question simple, we assume that the average effusion velocity of the radiation particles with positive energy along the radial is

$$v_e = \bar{v}_r^{(+)} = \int_0^\infty v_r f(v_r)\, \mathrm{d}v_r, \tag{11}$$

where the v_r is the radial velocity of the particles with positive energy, $f(v_r)$ is the radial velocity distribution function of the particles with positive energy, and the superscript $(+)$ expresses the integral scope $v_r > 0$. The value of v_e is not only related to the kinds of radiation particles, but also space–time metric near the event horizon. The greater the black hole mass is, the stronger the gravitational field near its event horizon is, the smaller the value of v_e is. Because the gravitational field near the event horizon is extremely strong, the value of v_e is very small. The average time which it takes for the particles with positive energy to reach the radiation spherical area of the black hole $4\pi(r_H + \varepsilon + \delta)^2$ is expressed by

$$\bar{t} = \frac{\delta\lambda}{2v_e}, \tag{12}$$

where λ is a revised constant. Combining (10) and (12), one can obtain the radiation energy flux of the scalar field near the black hole event horizon.

$$M_h = \frac{\rho V}{A\bar{t}} = \frac{4\pi^2 M^2 v_e}{15\varepsilon(\varepsilon+\delta)\lambda} T^4. \tag{13}$$

Because the r_H is much greater than ε and δ, in (13), one can adopt $A = 4\pi r_H^2$. Let

$$\sigma = \frac{4\pi^2 M^2 v_e}{15\varepsilon(\varepsilon+\delta)\lambda}. \tag{14}$$

(13) can be rewritten as

$$M_h = \sigma T^4. \tag{15}$$

It can be seen that the radiation energy flux of the black hole is proportional to the quartic of the temperature of its event horizon when ε, δ and v_e are all fixed. This result is similar to the Stefan–Boltzmann law of the blackbody radiation in flat space–time. Therefore, (15) can be called the generalized Stefan–Boltzmann law of the Schwarzschild black hole. And the (14) corresponds to the generalized Stefan–Boltzmann coefficient. Compared with the Stefan–Boltzmann law of blackbody radiation in flat space–time, σ is no longer a constant. When the cut-off distance and the thin film thickness are both fixed, it is a proportional coefficient which is related to the black hole mass, the kinds of radiation particles and space–time metric near the event horizon. The greater the black hole mass is, and the smaller the cut-off distance and thin film thickness are, the more significant the vacuum fluctuation is. Then the value of σ will be greater because Hawking radiation is related to the vacuum fluctuation near the event horizon. When the average effusion velocity of radiation particles along the radial is greater, their ability to escape from the black hole is stronger, and then the value of σ will be larger. This result is consistent with (14). From (13), one can obtain the radiation power of the black hole

$$P = \frac{64\pi^3 M^4 v_e}{15\varepsilon(\varepsilon+\delta)\lambda} T^4. \tag{16}$$

3 The Radiation Energy Flux Received by Observer Far Away from the Schwarzschild Black Hole

In order to study the effects of the gravitational field and the electromagnetic field around the radiation source in curved space–time on its thermal radiation, motivated by the particle model in classical mechanics, we introduce a thermal particle model of the radiation source in curved space–time. For a radiation source with the mass M, charge Q and the radiation temperature T, if its dimension is much smaller than other dimensions involved in research issues, that is to say, its size and shape are non-functional or only play a secondary role, this radiation source can be considered as a geometric point without size and shape, but it holds the mass, the charge and temperature of the whole radiation source. And then, we call such a geometric point a thermal particle. Although the thermal particle is only an ideal model, it can bring much convenience for us to study the thermal radiation of the object in curved space–time. For example, the distant star and the static spherically symmetric black hole can be considered as thermal particles under special circumstance. One can further study their thermal radiation and discover the effects of the gravitational field and electromagnetic field around them on their thermal radiation.

For the Schwarzschild black hole far away from an observer, it can be considered as a thermal particle. Then, from (13) and (14), one can obtain that the radiation energy flux received by the observer is

$$M_r = \frac{16\pi^2 M^4 v_e}{15\lambda\varepsilon(\varepsilon+\delta)r^2} T^4, \tag{17}$$

where r is the distance between the observer and the black hole. It can be seen that when the thin film thickness and the cut-off distance are both fixed, the radiation energy flux received by observer far away from the Schwarzschild black hole is proportional to the average radial effusion velocity of the radiation particles in the thin film, and inversely proportional to the square of the distance between the observer and the black hole. One can verify the following formulas by dimensional analysis

$$l_p = \sqrt{\frac{\hbar G}{c^3}} = 1.61 \times 10^{-35}\ \text{m}, \tag{18}$$

$$m_p = \sqrt{\frac{\hbar c}{G}} = 2.18 \times 10^{-8}\ \text{kg}, \tag{19}$$

$$\sigma_p = \frac{k_B^4}{\hbar^3 c^2} = 3.43 \times 10^{-7}\ \text{W} \cdot \text{m}^{-2} \cdot \text{K}^{-4}, \tag{20}$$

where l_p is the Planck length, m_p the Planck mass, σ_p the Planck generalized Stefan-Boltzmann coefficient, $\hbar$ the Planck constant, G the gravitational constant, c the light speed in vacuum, k_B the Boltzmann constant. By the (18)–(20), (14) can be written in the common unit system

$$\sigma = \frac{4\pi^2 G^2 k_B^4 M^2 v_e}{15\hbar^3 c^7 \lambda\varepsilon(\varepsilon+\delta)}. \tag{21}$$

In the common unit system, the relation between the temperature of the Schwarzschild black hole and its mass is

$$T = \frac{\hbar c^3}{8\pi k_B G M}. \tag{22}$$

Combining (21) and (22), (17) can be written in common unit system

$$M_r = \frac{\hbar c v_e}{3840\pi^2 \lambda\varepsilon(\varepsilon+\delta)r^2}. \tag{23}$$

It can be seen that when the cut-off distance and the thin film thickness are both fixed, the radiation energy flux of the black hole is proportional to the average effusion velocity of the radiation particles, and inversely proportional to the square of the black hole mass.

From (23), in common unit system, the radiation power of scalar field of the Schwarzschild black hole is

$$P = \frac{\hbar c v_e}{960\pi\lambda\varepsilon(\varepsilon+\delta)}. \tag{24}$$

It can be seen that when the cut-off distance and the thin film thickness are both fixed, the radiation power of the Schwarzschild black hole is proportional to the average effusion velocity of the radiation particles.

4 Conclusion

By the entropy density near the event horizon, the thermal radiation law of the black hole is studied and the result is obtained that thermal radiation of the black hole satisfies the generalized Stefan–Boltzmann law. The derived generalized Stefan–Boltzmann coefficient is no longer a constant. When the cut-off distance and the thin film thickness are both fixed, it is a proportional coefficient which is related to the black hole mass, the kinds of radiation particles and the space–time metric near the event horizon. This research discovers the intrinsic relation between the gravitational field of the black hole and its thermal radiation. The thermal particle model is put forward, and by this model the thermal radiation of the Schwarzschild black hole is studied. These results show that when the cut-off distance and the thin film thickness are both fixed for the Schwarzschild black hole, the radiation energy flux received by the observer far away from the Schwarzschild black hole is proportional to the average radial effusion velocity of the radiation particles in thin film, and inversely proportional to the square of the distance between the black hole and the observer. Although one can obtain more thermal properties of the black hole by the thin film model, the cut-off factor can not be avoided, which denotes the limitations of the semi-classical approximation on the black hole entropy. The paper puts forward a new method to study the thermal radiation of the black hole.

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References

1. Bekenstein, J.D.: Phys. Rev. D **7**, 2333 (1973)
2. Hawking, S.W.: Commun. Math. Phys. **43**, 199 (1975)
3. 't Hooft, G.: Nucl. Phys. B **256**, 727 (1985)
4. Hochberg, D., Kephart, T.W., York, J.W.: Phys. Rev. D **48**, 479 (1993)
5. Li, X.: Phys. Lett. B **540**, 9 (2002)
6. Cognola, G., Lecca, P.: Phys. Rev. D **57**, 1108 (1998)
7. Li, X., Zhao, Z.: Phys. Rev. D **62**, 104001 (2000)
8. Gao, C.J., Liu, W.B.: Int. J. Theor. Phys. **39**, 2221 (2000)
9. Li, X., Zhao, Z.: Chin. Phys. Lett. **18**, 463 (2001)
10. Liu, W.B., Zhao, Z.: Chin. Phys. Lett. **18**, 310 (2001)
11. He, F., Zhao, Z.: Phys. Rev. D **64**, 044025 (2001)
12. He, H., Zhao, Z., Zhang, L.H.: Int. J. Theor. Phys. **41**, 1781 (2002)
13. Liu, W.B., Zhao, Z.: Int. J. Mod. Phys. A **16**, 3793 (2001)
14. Gibbons, G.W., Hawking, S.W.: Phys. Rev. D **15**, 2752 (1977)
15. Parikh, M.K., Wilczek, F.: Phys. Rev. Lett. **85**, 5024 (2000)
16. Zhang, J.Y., Zhao, Z.: Phys. Lett. B **618**, 14 (2005)

17. Jiang, Q.Q., Wu, S.Q., Cai, X.: Phys. Rev. D **73**, 064003 (2006)
18. Zhang, J.Y.: Phys. Lett. B **668**, 353 (2008)
19. Jiang, J.J., Meng, Q.M., Wang, S.: Int. J. Theor. Phys. **48**, 2826 (2009)
20. Jiang, Q.Q., Wu, S.Q.: Phys. Lett. B **635**, 151 (2006)
21. Meng, Q.M.: Acta Phys. Sin. **52**, 2102 (2003) (in Chinese)
22. Meng, Q.M.: Acta Phys. Sin. **54**, 471 (2005) (in Chinese)
23. Meng, Q.M., Su, J.Q., Jiang, J.J.: Acta Phys. Sin. **56**, 5077 (2007) (in Chinese)
24. Meng, Q.M., Jiang, J.J.: Sic. China Ser. G **51**, 923 (2008)
25. Meng, Q.M., Wang, S., Jiang, J.J., Deng, D.L.: Chin. Phys. B **17**, 2811 (2008)
26. Meng, Q.M., Jiang, J.J., Liu, J.L., Deng, D.L.: Acta Phys. Sin. **58**, 78 (2009) (in Chinese)
27. Jiang, J.J., Meng, Q.M., Wang, S.: Chin. Phys. B **18**, 457 (2009)
28. Meng, Q.M., Jiang, J.J., Wang, S.: Acta Phys. Sin. **58**, 7486 (2009) (in Chinese)